CHAPTER 1: MATHEMATICS OF INVESTMENT

Why do you need to know investments, bonds, stocks, interests? Why is there a need to invest your hard earned money?

Whether you just want to save for that phone or tablet that you wanted to buy; or you’re saving for a gift that you wanted to give to your parent’s birthday; or you as a parent planning on using your credit card in paying your child’s tuition fee payable in 6 months with 0% interest; all these reasons why you plan invest or save your money, entails knowing some things about interest rates, and ideas in investment or finance. Studying the mathematics behind finance and investment

Having the knowledge in basic concepts in business mathematics or the mathematics of investment may help you decide whether to use that credit card for a 5% interest compounded monthly or a simple interest for a period of 6 months. Some topics might shed light on which banks would give a higher interest rate for your savings. As a young couple starting a family, one might plan for their children’s future by understanding stocks and bonds or fund accumulations. These are but a few reasons for investing your hard earned money.

In this chapter, we will discuss the concepts of simple and compound interest as well as simple and general annuities. The notions of stocks and bonds will be viewed as a simple financial instrument.
1.1 Simple Interest

In financial transactions an **interest** is the amount paid by a borrower to a lender for the use of money over a period. Interest that is paid as a percent of amount borrowed or invested is called **simple interest**. The formula for simple interest is given by the following:

**Simple Interest**

\[
I = Prt
\]

Where,

\[
I = \text{interest earned (owed)}
\]

\[
P = \text{principal amount invested (or borrowed)}
\]

\[
r = \text{annual rate of interest}
\]

\[
t = \text{term or time frame in years}
\]

**Example 1.** Suppose Kiko wanted to invest an amount *Php 50,000.00* for 2 years at a financial institution that gives a simple interest of 3% per year. The interest rate was given to Kiko by the financial institution on the assumption that he cannot withdraw the investment within the 2-year period. How much is Kiko’s earning on the investment after the 2-year period?

**Solution:**

The following can be obtained from the problem: \( P = 50,000, r = 0.03, t = 2 \).

\[
I = Prt = (50,000)(0.03)(2) = 3,000
\]

From this we conclude that, the investment earned * Php3,000.00.*
Interest can be viewed as a lender or a borrower. Sometimes if we are the investor, we consider the value of our investment after a given period. In this case we introduce the concept of **future values** or **accumulated values** or **maturity value**.

### Future Value

\[
F = P + I_s
\]
\[
F = P(1 + rt)
\]

Where,

- \( I_s = \text{interest earned (owed)} \)
- \( P = \text{principal amount invested (or borrowed)} \)
- \( r = \text{annual rate of interest} \)
- \( t = \text{term or time frame in years} \)

**Example 2.** April wants to borrow \( Php40,000.00 \) from a bank that gives an annual interest rate of 4.5%. However, she only wants to borrow the fund for a 9-month period and will be able to pay the bank immediately after 9 months. How much interest is she going to pay from borrowing the amount of money? What is the accumulated value of the amount borrowed after the 9-month period?

**Solution:**

The following can be obtained from the problem: \( P = 40,000 \), \( r = 0.045 \), \( t = 0.75 \) since she only borrowed the fund for 9-months which is \( \frac{3}{4} \) of a year.

\[
I_s = Prt = (40,000)(0.045)(0.75) = 1,350.
\]

From this we conclude that, the interest due is \( Php1,350.00 \).
In this example the accumulated value of the amount borrowed is \( F = P + I_s \) that is; the sum of the principal amount or the amount borrowed and the interest. Thus, after nine months, April will pay the bank \( \text{Php} \ 41,350.00 \)

**Example 3.** What is the simple interest rate applied if an investment of \( \text{Php}37,500 \) accumulates to \( \text{Php}41,812.50 \) in the period of 5 years?

**Solution:**
We note that the interest earned by the investment is \( \text{Php}4312.5 \) that is, \( I = 4312.5 \). From the formula \( I = Prt \), we have \( r = \frac{I}{Pt} = \frac{4312.5}{(37,500)(5)} = 0.023 = 2.3\% \)

**Example 4.** The repayment on a loan was \( \text{Php}16,275 \). If the loan was for 15 months or 1.25 years at 6.8% interest a year, how much was the principal?

**Solution:**
Based from the given we have the following: \( F = 16,275 \), \( r = 0.068 \), and \( t = 1.25 \)
Since \( F = P(1 + rt) \), we have \( P = \frac{F}{1 + rt} = \frac{16,275}{1 + (0.068)(1.25)} = 15,000 \).
Different ways of expressing time/term of a loan or investment.

Sometimes the term of investment is not given in years. The term or time frame given in certain problems maybe stated in days or months. In cases where the time is expressed in months it is easy to express it in years. But when the term/time is given in days we use a time factor such as the following:

\[
\begin{align*}
\text{Actual time} & \quad \text{Ordinary Simple Interest or Bankers Rule} \\
& \quad \frac{\text{Actual time}}{360} \\
\text{Actual time} & \quad \text{Exact Simple Interest} \\
& \quad \frac{\text{Actual time}}{365} \\
\text{Approximate time} & \quad \text{Approximate time} \\
& \quad \frac{\text{Approximate time}}{360} \\
& \quad \frac{\text{Approximate time}}{365}
\end{align*}
\]

*Actual time* – Number of days until the repayment date except the origin date.

Remark

The Bankers Rule or Ordinary Simple Interest is applied whenever a given problem does not specify the time factor to be used.

Example 5. Find the actual and approximate time from May 1, 1983 to September 15, 1983.

<table>
<thead>
<tr>
<th>Actual Time</th>
<th>Approximate Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>May</td>
<td>May</td>
</tr>
<tr>
<td>June</td>
<td>June</td>
</tr>
<tr>
<td>July</td>
<td>July</td>
</tr>
<tr>
<td>Aug</td>
<td>Aug</td>
</tr>
<tr>
<td>Sept</td>
<td>Sept</td>
</tr>
<tr>
<td>30</td>
<td>29</td>
</tr>
<tr>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>31</td>
<td>30</td>
</tr>
<tr>
<td>31</td>
<td>30</td>
</tr>
<tr>
<td>15</td>
<td>15</td>
</tr>
<tr>
<td>137</td>
<td>134</td>
</tr>
</tbody>
</table>
Example 6. Find the actual and approximate time from April 15, 2008 to December 21, 2008.

<table>
<thead>
<tr>
<th>Actual Time</th>
<th>Approximate Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>April</td>
<td>15</td>
</tr>
<tr>
<td>May</td>
<td>30</td>
</tr>
<tr>
<td>June</td>
<td>30</td>
</tr>
<tr>
<td>July</td>
<td>31</td>
</tr>
<tr>
<td>Aug</td>
<td>31</td>
</tr>
<tr>
<td>Sept</td>
<td>30</td>
</tr>
<tr>
<td>Oct</td>
<td>31</td>
</tr>
<tr>
<td>Nov</td>
<td>30</td>
</tr>
<tr>
<td>Dec</td>
<td>21</td>
</tr>
<tr>
<td></td>
<td>250</td>
</tr>
</tbody>
</table>

30 - 15 = 15

Example 7. Find the actual and approximate time from June 25, 2008 to Nov 18, 2008.

<table>
<thead>
<tr>
<th>Actual Time</th>
<th>Approximate Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>June</td>
<td>5</td>
</tr>
<tr>
<td>July</td>
<td>31</td>
</tr>
<tr>
<td>Aug</td>
<td>31</td>
</tr>
<tr>
<td>Sept</td>
<td>30</td>
</tr>
<tr>
<td>Oct</td>
<td>31</td>
</tr>
<tr>
<td>Nov</td>
<td>18</td>
</tr>
<tr>
<td></td>
<td>146</td>
</tr>
</tbody>
</table>

30 - 25 = 5
EXERCISES

1. Determine the Actual and Approximate number of days in the given origin and repayment dates.

<table>
<thead>
<tr>
<th>Origin Date</th>
<th>Repayment Date</th>
<th>Actual Time</th>
<th>Approximate Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 22, 1995</td>
<td>July 09, 1995</td>
<td></td>
<td></td>
</tr>
<tr>
<td>January 06, 1997</td>
<td>November 06, 1997</td>
<td></td>
<td></td>
</tr>
<tr>
<td>March 03, 2007</td>
<td>October 11, 2007</td>
<td></td>
<td></td>
</tr>
<tr>
<td>February 04, 1990</td>
<td>November 05, 1992</td>
<td></td>
<td></td>
</tr>
<tr>
<td>March 02, 2005</td>
<td>November 05, 2006</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2. Joseph borrowed Php 5,000 on November 2, 2015 from Arthemus, which is to be repaid on May 21, 2016 at 6.2% simple interest per year. Find the amount to be repaid. How much will the interest be at the repayment date if the following time factors are used?
   a. Bankers Rule
   b. Exact Simple Interest
   c. $\frac{\text{Approximate}}{360}$
   d. $\frac{\text{Approximate}}{365}$

3. How much should Mark pay to Michele if he borrowed Php 10,000 on June 25, 2015 and if the principal and interest are to be paid on November 18, 2015 at 15% simple interest per year? Use the following time factors.
   a. Bankers Rule
   b. Exact Simple Interest
   c. $\frac{\text{Approximate}}{360}$
   d. $\frac{\text{Approximate}}{365}$
4. At what simple interest rate will a sum of money double itself in 5 years?

5. If Wendy wants to invest her Php25,000, how many years will it take for her savings to accumulate to Php 40,000 if she invested her savings to a financial institution that provides a simple interest rate is 4.5% per year?

6. An amount of P12, 500 is invested at 3.25% simple interest for 3 years. Complete the table below.

<table>
<thead>
<tr>
<th>Time (in years)</th>
<th>Principal</th>
<th>Rate</th>
<th>Interest</th>
<th>Future Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>P12, 500</td>
<td>0.0325</td>
<td>P0</td>
<td>P0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

7. What will be the maturity value of P15, 500 if it is borrowed at 10.5% rate for 10 months?

8. How much should Mrs. Dolores invest today in a time deposit with 5.5% interest if she expects to have P175, 000 for his son’s education at the end of 5 years?

9. Mr. Pascual, an airline owner, decided to invest P2.5 million to fund his department handling spare parts replacement. How long will it take his investment to accumulate to P3.325 million if the bank’s interest rate is 5.5%?

10. FJC Printing would like to invest a certain amount in a bank that will accumulate to P157, 800 in 550 days for the replacement of a printer. If the bank offers 8% interest, how much must be invested at the start of the term under

11. Use Banker’s Rule to compute the simple interest of P10, 000 investment at 10% simple interest rate from April 14, 2004 to November 18 of the same year.

12. The university treasurer puts half a million pesos to a time deposit offering 7% for 2 years. How much is in the fund at the end of the term?

13. What rate was applied to a 4-year loan of P278, 000 in which the maturity value is P372, 520?
1.2 Compound Interest

Consider an investment whose time frame is divided into equal intervals. If an interest is computed after an interval and is being added to the principal and thereafter earns an interest, then the difference between the original principal and the total amount after the whole time frame is called **compound interest**. The **compound amount** or the **accumulated value** of the principal is the sum of the principal and the compound interest. In this situation, we see that the interest is being converted into a principal and thus we use the phrase “interest is compounded” or “interest is converted”.

Consider an investment amount $P$ placed on a financial institution that gives a compound interest where the interest rate per conversion period is $i$. After one conversion, the total amount due to the investor is $P + Pi$. The new principal at the end of the first conversion is now $P + Pi = P(1 + i)$. At the end of the second period, the accumulated value now becomes, $P + Pi + (P + Pi)i$ which is equivalent to $P + Pi + Pi + Pi^2 = P(1 + i)^2$. Thus, the new principal after the second conversion is $P(1 + i)^2$. In a similar manner, at the end of the third period or third conversion, the accumulated value becomes $P(1 + i)^2 + P(1 + i)^2i = P(1 + i)^2(1 + i) = P(1 + i)^3$. Now, following the patterns that we see, after the $n^{th}$ conversion, the accumulated value becomes $P(1 + i)^n$. Thus we have the following formula:

$$F = P(1 + i)^n$$

Where,
- $F$ = compound amount or accumulated value of $P$ at the end of the term
- $P$ = present value or original principal
- $j$ = interest rate per year
- $i$ = interest rate per period
- $n$ = total number of conversions periods
- $m$ = number of conversions per year
- $t$ = term

$\frac{i}{m} = j$  \hspace{1cm} $n = tm$
In the context of compound interest, the interest rate per annum or per year is called the nominal rate of interest. Thus when a given nominal rate is said to be compounded quarterly, that means in a given year there will be 4 conversions. Similarly, when we say compounded monthly, the conversions are made every month therefore in a given year, there will be 12 conversions.

Example  Find the compound amount and interest:

a. If Php 2,500 is invested at 13% compounded quarterly for 12 years

Solution

Given: \( P = 2,500; \ j = 13\%; \ t = 12; \ m = 4 \)

\[ n = tm = 4 \times 12 = 48 \]

\[ i = \frac{j}{m} = \frac{0.13}{3} = 0.0325 \]

\[ F = P(1 + i)^n = 2,500(1.0325)^{48} = Php 11,605.47 \]

b. If Php 3,700 is invested at 12% compounded semi-annually for 5 years

Solution

Given: \( P = 3,700; \ j = 12\%; \ t = 5; \ m = 2 \)

\[ n = tm = 2 \times 5 = 10 \]

\[ i = \frac{j}{m} = \frac{0.12}{2} = 0.06 \]

\[ F = P(1 + i)^n = 3,700(1.06)^{10} = Php 6,626.14 \]

Example  Find the present value of Php 2,850 due in 5 years if money is worth 10% compounded quarterly.

Solution

Given: \( F = 2,850; \ j = 10\%; \ t = 5; \ m = 4 \)
Example  How much must be invested today in a savings account to realize Php 9,000 in 4 years, if money earns at the rate of 4% compounded quarterly?

Solution
Given: \( F = 9,000 \); \( j = 4\% ; t = 4; m = 4 \)
\( n = tm = 4 \times 4 = 16 \)
\( i = \frac{j}{m} = \frac{0.04}{4} = 0.01 \)
\[ P = \frac{F}{(1 + i)^n} = 9,000(1.01)^{-16} = \text{Php 7,675.39} \]

Example  What rate compounded annually will double any amount principal if it is invested in 6 years?

Solution
Let \( x \) be the amount to be invested. \( m = 1; t = 6 \Rightarrow n = 6 \). We want to find \( j \).

\[ F = P(1 + i)^n \Rightarrow 2x = x\left(1 + \frac{j}{1}\right)^6 \Rightarrow 2 = (1 + j)^6 \Rightarrow j = \sqrt[6]{2} - 1 \Rightarrow j = 12.25\% \]
We want to find \( j \). Given: \( m = 2; t = 8.5 \Rightarrow n = 17 \).

\[
F = P(1 + i)^n \Rightarrow 50,000 = 20,000 \left( 1 + \frac{j}{2} \right)^{17} \Rightarrow 2.5 = \left( 1 + \frac{j}{2} \right)^{17} \Rightarrow j = 2\left(\sqrt[17]{2.5} - 1\right) \Rightarrow j = 11.08\%
\]

**Example**  When will Php 30,000 earn interest of Php 15,000 if it is invested at the rate of 7.5% converted annually?

**Solution**
We want to find \( t \). Given: \( F = 45,000; P = 30,000; j = 7.5\%; m = 1 \)

\[
F = P(1 + i)^n \Rightarrow 45,000 = 30,000 \left( 1 + \frac{0.075}{1} \right)^n \Rightarrow 1.5 = (1 + 0.075)^n \Rightarrow \log 1.5 = n \log (1.075) \Rightarrow n = \frac{\log 1.5}{\log (1.075)} \Rightarrow n = 5.6065 \Rightarrow t = 5.6065
\]

**Example**  When will a principal double itself if the interest rate is 14% compounded quarterly?

**Solution**
We want to find \( j \). Given: \( F = 2P; j = 14\%; m = 4 \)

\[
F = P(1 + i)^n \Rightarrow 2P = P \left( 1 + \frac{0.14}{4} \right)^n \Rightarrow 2 = (1.035)^n \Rightarrow \log 2 = n \log (1.035) \Rightarrow n = \frac{\log 2}{\log (1.035)} \Rightarrow n = 20.1488 \Rightarrow t = 5.0372
\]

Sometimes we may want to compare which interest rate would provide a higher interest when their interest payments are not the same. We then have to resort to converting these interest rates to a common interest payment. This is the notion of effective rates of interest. For instance, which interest rate gives a higher interest for an investment of 1 Peso (Php1), an 8%
compounded semi-annually or a 7.9% compounded monthly. If 1 peso is invested at a rate of 8%
compounded semi-annually, then at the end of the year it accumulated to \( S = 1 \left( 1 + \frac{0.08}{2} \right)^2 = 
1.0816 \). While an investment of 1 peso at a rate of 7.9% compounded monthly accumulates to
\( S = 1 \left( 1 + \frac{0.08}{12} \right)^{12} = 1.082999507 \).

The effective rate of interest of \( i \) compounded \( m \) times a year can be computed as
\( r = (1 + i)^m - 1 \)

**Example**

Determine the effective rate of interest for each of the following nominal interest rate \( j \)
compounded \( m \) times a year.

<p>| | | | |</p>
<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( i = \frac{j}{m} )</td>
</tr>
<tr>
<td>2%</td>
<td>4</td>
<td>0.5%</td>
<td>(1 + 0.5%)^4 - 1 = 4.0625%</td>
</tr>
<tr>
<td>3%</td>
<td>3</td>
<td>1%</td>
<td>(1 + 1%)^3 - 1 = 7%</td>
</tr>
<tr>
<td>1.5%</td>
<td>12</td>
<td>0.125%</td>
<td>(1 + 0.125%)^{12} - 1 = 3.10989%</td>
</tr>
<tr>
<td>5%</td>
<td>4</td>
<td>1.25%</td>
<td>(1 + 1.25%)^{4} - 1 = 24.6289%</td>
</tr>
</tbody>
</table>

**EXERCISES**

1. Mary Joy deposited P14,500 in a bank that pays interest at 2% compounded annually. If
no withdrawal is made, how much does she have in her account after 5 years?

2. If the principal invested by Anika is P50,000 and the interest rate given by Peso Financial
Inc. is 2.5% compounded quarterly, how much did she earned at the end of 5 years?

3. Find the compound interest earned at the end of 2 year and 5 months of an investment
fund amounting to P24,500 if it is invested at 3.5% compounded monthly.
4. If money can be invested at 7% compounded monthly, find the present value of 55,300 which is due after 2 years and 11 months from today.

5. Jamie wants to have P45,000 in 2 years to buy a new computer. How much money should he invest today in a fund that earns 5% compounded quarterly to get this amount after 2 years?

6. A 35,000 principal earned an interest of P8,500 at the end of 7 years. At what nominal rate, compounded annually, was it invested?

7. At what rate compounded monthly should P25,000 be deposited in a bank to gain an interest of P4,500 in 3 years?

8. If P48,000 is invested at the rate of 2.5% compounded quarterly, when will the compound amount be P70,000?

9. When will P80,000 grow to P95,000 if it is invested at 4.5% compounded quarterly?

10. If P135,650 is the maturity value of a sum invested at 3.2% compounded semi-annually for 9 years and 6 months, find the present value and the compound interest earned.

1.3 Annuities

An **Annuity** is a sequence of equal payments made at equal periods or time intervals. These payments may be made annually semi-annually, quarterly or at other periods. Some examples are:

- Monthly payments of rent
- Monthly wages
- Annual premiums on a life insurance policy
- Quarterly payments for a car loan
• Annual payments on a bond

In our study of annuities, we need to be familiar with the following terms: payment interval, term, periodic payment, simple annuity.

**DEFINITION**

The **payment interval** is the time between successive payments of an annuity. The **term** of an annuity is the time between the first payment interval and the last payment interval. The **periodic payment**, denoted by $R$, is the amount of each payment. A **Simple annuity** is an annuity in which the payment interval is the same as the interest period. If the payment intervals are not the same as the interest period, then the annuity is called a **general annuity**.

**Example** 16. A four-year lease agreement between Alfred and Thrifty Mall Inc. (TMI) indicates that, Alfred pays TMI Php 100,000 at the end of every year if the agreed interest rate is 5% compounded quarterly.

In this example, the payment period is a whole year. However, the interest period is quarterly or every 3 months. Hence, the annuity is a general annuity.

**Example** 16. Remy needs to repay her debt to Marvin by paying Php 150 at the end of every 6 months for 4 years in a bank account that gives an interest rate of 4.35% compounded semi-annually. This is an example of a simple annuity since the payment intervals are the same as the interest period.
There are three types of simple annuities and they are Ordinary Annuities, Annuity Dues and Deferred Annuities. These annuities differ only by the date of the first regular payment and hence, it would be natural to see a relationship between the formulas for the accumulated value or present value of an annuity compared to the other.

**DEFINITION**

An **Ordinary Annuity** is an annuity in which the payments are made at the end of each payment interval.

The sum or accumulated value of the annuity or amount of an ordinary annuity, denoted by $S$, is given by

\[
S = R + R(1+i) + R(1+i)^2 + \cdots + R(1+i)^{n-2} + R(1+i)^{n-1} = R \left[ \frac{(1+i)^n - 1}{i} \right]
\]

The present value of an ordinary annuity, denoted by $A$ is just the sum of the present value of each of the payments and is illustrated below:
Example 17. Michele wants to venture into the food business stands. She plans to create a fund by making deposits of Php 30,000 at the end of every 6 months in a bank that gives 4% interest compounded semi-annually, how much money will be in the fund after 4 years?

Solution:
In this example we see that the regular payments are $R = Php 30,000$ and the interest rate per period is $i = 0.04/2 = 0.02$. Since payments are made twice a year for 4 years, the total number of payments is $n = 8$. Thus to get the accumulated value of the annuity we have

$$S = R \left[ \frac{(1 + i)^n - 1}{i} \right] = 30,000 \left[ \frac{(1 + 0.02)^8 - 1}{0.02} \right] = Php 257,489.07$$

**Example** A father wants to invest Php 1.5M in a financial institution that gives 8% compounded monthly so that he could give a monthly allowance for his daughter for the next 15 years. How much will the monthly allowance be?

**Solution:**

In this example, the monthly allowance for 15 years is unknown and that the present value of these allowances should amount to Php 1.5M where the rate of interest $i = \frac{0.08}{12} = \frac{1}{150}$ and $n = (12)(15) = 180$.

$$A = R \left[ \frac{1 - (1 + i)^{-n}}{i} \right] \rightarrow$$

$$1,500,000 = R \left[ \frac{1 - (1 + \frac{1}{150})^{-180}}{\frac{1}{150}} \right] \rightarrow$$

$$R = \frac{1,500,000}{\frac{1 - (1 + \frac{1}{150})^{-180}}{\frac{1}{150}}} = Php 14,334.78$$

Thus, the monthly allowance of the daughter from based from the investment is Php 14,334.78.

**Example**

Mario wants to accumulate Php 100,000 by making deposits of Php 500 at the end of every month in a cooperative bank that gives 5% interest rate compounded monthly. At least how many payments should he make to the bank to attain his investment goal?

**Solution:**

The following are given in the problem: $S = 100,000, R = 500, i = \frac{0.05}{12}$ and we wish to find $n$. 

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Prepared by: Francis Joseph H. Campeña
In the formula, \( S = R \left[ \frac{(1+i)^n-1}{i} \right] \), solving for \( n \) gives us \( n = \frac{\log[S/i]}{\log[1+i]} \) thus plugging in the known values we obtain \( n \approx 145.78 \approx 146 \). Mario should at least make 146 deposits amounting to Php 500 each to accumulate Php 100,000 in the said bank.

However, sometimes we want to accumulate a certain amount of money by saving or depositing a regular payment within a specified time frame. We then look for banks that can provide such an investment scheme and thus we need to check which banks offers an interest rate that will accommodate the given situations. If \( A \) is given, then solving for \( i \) in the equation \( (n^2 - 1)i^2 + 6(n + 1)i + 12 \left( 1 - \frac{nR}{A} \right) = 0 \), provides us that answer. However, if \( S \) is given, then solving for \( i \) in the equation \( (n^2 - 1)i^2 - 6(n - 1)i + 12 \left( 1 - \frac{nR}{S} \right) = 0 \) gives us the answer.

These equations are of quadratic in form and the use of a quadratic formula to solve for \( i \) leads us to

\[
i = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Lastly to solve for the nominal rate, we have

\[ j = im \]

Where \( m \) is the number of conversions per year.

**Example**

John bought a car worth P984,000. He paid P90,000 cash and agreed to pay P15,500 at the end of every month for 10 years. At what nominal rate compounded monthly was he charged?

**Solution:**

The following are given in the problem: \( A = 894,000, R = 15,000, t = 10, m = 12 \) and we wish to find \( j \).

\[
(120^2 - 1)i^2 + 6(120 + 1)i + 12 \left( 1 - \frac{120(15000)}{894,000} \right) = 0
\]

\[
14399i^2 + 726i - 12.1611 = 0
\]
We now use quadratic formula to solve for $i$.

$$i = \frac{-726 \pm \sqrt{726^2 - 4(14399)(-12.1611)}}{2(14399)}$$

Since the other value of $i$ is -0.064 which is not possible, we have $i = 0.0133$. The nominal rate is $j = (0.0133)(12) = 0.1596 = 15.96\%$.

**Example**

Sheena wanted to save P500,000 in 15 years. She was proposed by a fund manager to invest P7,500 every quarter at Sigma Investment Group. Find the interest rate given by Sigma Investment Group.

**Solution:**

The following are given in the problem: $S = 500,000, R = 7,500, t = 15, m = 4$ and we wish to find $j$.

$$(60^2 - 1)i^2 - 6(60 - 1)i + 12 \left(1 - \frac{(60)(7,500)}{500,000}\right) = 0$$

$$3599b i^2 - 354i + 1.2 = 0$$

We now use quadratic formula to solve for $i$.

$$i = \frac{114 \pm \sqrt{(-114)^2 - 4(399)(-84)}}{2(399)}$$

We obtain two values of $i$, which are 0.0948 and 0.0035. To determine which is the right interest rate, we have to solve of $S$ using both values.

When $i = 0.0948$, $S = 7500 \left(\frac{(1 + 0.0948)^{60} - 1}{0.0948}\right) = 18,048,483.6$

When $i = 0.0035$, $S = 7500 \left(\frac{(1 + 0.0035)^{60} - 1}{0.0035}\right) = 499,769.62$
Note: Since we rounded off the values of \( i \), we will not get the exact value of P500,000.

From the computed values of \( S \), the nominal rate given by Sigma Investment Group is 1.4%.

Example

Earl is interested in buying a lot worth P2,500,000. He agrees to give a down payment of P500,000 and to pay P23,500 every month for as long as necessary. He will be charged an interest of 8.5% monthly.

a. How many monthly payments of P23,500 must he make?

b. How much is the final payment if it is to be made on the same day as the last P23,500 payment?

c. How much is his final payment if it is to be made one month after the last P23,500 payment to completely pay off the lot?

Solution:

The following are given in the problem: \( = 2,000,000 \), \( R = 23,500 \), \( i = \frac{0.085}{12} \).

a. In the formula, \( A = R \left[ \frac{1-(1+i)^{-n}}{i} \right] \), solving for \( n \) gives us \( n = \frac{\log \left[ \frac{1-A}{R} \right]}{\log(1+i)} \).

We have \( n = \frac{\log \left[ \frac{1-(2,000,000)(0.085/12)}{23,500} \right]}{\log(1+0.085/12)} = 130.82 \). This means that Earl will be making 130 full payments of P23,500.

b. Given that \( n=130.82 \), it tells us that we don’t need 131 full payments of P23,500. We must compute for the additional amount to be paid on the 130\(^{th}\) month. We will be using the 130\(^{th}\) month as the comparison date. We now solve the value of 2,000,000 and total payment at the 130\(^{th}\) month.

\[
2,000,000 \left( 1 + \frac{0.085}{12} \right)^{130} = 5,006,487.38
\]
\[ S_{130} = 23,500 \left( \frac{1 + \frac{0.085}{12}}{\frac{0.085}{12}} \right)^{130} - 1 \] = 4,987,232.01

The additional amount will be 5,006,487.38 − 4,987,232.01 = 19,255.37.

The final payment will be P42,755.37.

c. Using the value computed from (b), the final payment to be made one month after the last P23,500 payment is 19,255.37 \left( 1 + \frac{0.085}{12} \right) = 19,391.76

---

**DEFINITION**

An **Annuity Due** is an annuity in which the payments are made at the beginning of each payment interval.

\[ \hat{S} = R(1+i)^{n+1} + R(1+i)^n + R(1+i)^{n-1} + \cdots + R(1+i)^2 + R(1+i)^0 = R \left[ \frac{(1+i)^{n+1} - 1}{i} \right] - R \]
The above formula for the accumulated value of an annuity due is equivalent to
\[
\tilde{S} = R \left[ \frac{(1 + i)^{n+1} - 1}{i} \right] = R \left[ \frac{(1 + i)^{n+1} - 1 - i}{i} \right] = R \left[ \frac{(1 + i)^{n+1} - (1 + i)}{i} \right]
\]
\[
= R \left[ \frac{(1 + i)^n - 1}{i} \right] (1 + i).
\]
The above formula shows the relationship of the accumulated value of an ordinary annuity \(S\) and the accumulated value of an annuity due, \(\tilde{S}\); that is \(\tilde{S} = S(1 + i)\).

The present value of an annuity due, denoted by \(\mathcal{A}\) is just the sum of the present value of each of the payments and is illustrated below:

\[
\mathcal{A} = R + A_1 + A_2 + \cdots + A_{n-1} = R + R(1 + i)^{-1} + R(1 + i)^{-2} + \cdots + R(1 + i)^{-(n-1)} = R + R \left[ \frac{1 - (1 + i)^{-(n-1)}}{i} \right]
\]

Equivalently, we have
\[
\mathcal{A} = R \left[ 1 + \frac{1 - (1 + i)^{-(n-1)}}{i} \right] = R \left[ \frac{1 - (1 + i)^{-(n-1)}}{i} \right] = R(1 + i) \left( \frac{1 - (1 + i)^{-n}}{i} \right)
\]

**Example** A father wants to set up an educational fund at Thrift Bank of the Philippines for his daughter by making a deposit of Php 10,000 at the beginning of every quarter of the year for 10 years. If money is worth 2.5% in Thrifty Bank of the Philippines, how much money is in the fund at the end of 10 years?
CHAPTER 1: MATHEMATICS OF INVESTMENT

Solution:

The annuity formed by the deposits is an annuity due with regular payments \( R = 10,000 \), \( n = 40 \) and \( i = \frac{0.025}{4} = 0.00625 \). We then have the following:

\[
\ddot{S} = R \left[ \frac{(1 + i)^n - 1}{i} \right] (1 + i)
\]

\[
= 10,000 \left[ \frac{(1 + 0.00625)^{40} - 1}{0.00625} \right] (1.00625)
\]

\[
= \text{Php} 459,325.14
\]

Thus, at the end of 10 years, the fund will be \( \text{Php} 459,325.14 \).

Example: A smart phone can be bought by making a cash payment of P5,000 and 6 quarterly payments of P4000 each, the first of which is due on the date of purchase. Find the cash value of the smart phone if money is worth 6% compounded quarterly.

Solution:

The following are given in the problem: \( R = 4,000 \), \( i = \frac{0.06}{4} = 0.015 \), \( n = 6 \).

\[
\dot{A} = R(1 + i) \left( \frac{1 - (1 + i)^{-n}}{i} \right)
\]

\[
= 4,000 (1.015) \left[ \frac{1 - (1.015)^{-6}}{0.015} \right]
\]

\[
= 23,130
\]

\[
CV = \dot{A} + DP
\]

\[
= 23,130 + 4,000
\]

\[
= \text{Php} 27,130.
\]
**Example**  Joshua wants to visit Japan 3 years from now. He will be needing a sum of P100,000 for his trip. How much must he put aside in his travel funds every year starting now if money is worth 6% compounded annually?

Solution:

The following are given in the problem: \( \ddot{s} = 100,000 \), \( i = 0.06 \) \( t = 3 \).

\[
\ddot{s} = R \left[ \frac{(1 + i)^n - 1}{i} \right] (1 + i)
\]

\[
100,000 = R \left[ \frac{(1+0.06)^3 - 1}{0.06} \right] (1.06)
\]

\[
R = \frac{100,000 \left[ \frac{(1+0.06)^3 - 1}{0.06} \right] (1.06)}{(1.06)}
R = 29,633
\]

---

**DEFINITION**

A **Deferred Annuity** is an ordinary annuity in which the first payment is made at some later date.
The sum or accumulated value of a deferred annuity denoted by $S_d$ is equivalent to that of the accumulated value of an ordinary annuity. As for the present value of a deferred annuity, we use the notation $A_d$ and is computed as $A_d = A(1 + i)^{-d}$ where $A$ is the present value of an ordinary annuity, $i$ is the interest rate applied for every interval and $d$ is the number of deferred payment intervals or period.

We can see that the accumulated value and present value of the three types of simple annuities are related as shown below.

<table>
<thead>
<tr>
<th></th>
<th>Ordinary Annuity</th>
<th>Annuity Due</th>
<th>Deferred Annuity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Present Value</strong></td>
<td>$S = R \left[ \frac{(1 + i)^n - 1}{i} \right]$</td>
<td>$\bar{S} = R \left[ \frac{(1 + i)^n - 1}{i} \right] (1 + i)$</td>
<td>$S_d = S$</td>
</tr>
<tr>
<td><strong>Accumulated Value</strong></td>
<td>$A = R \left[ \frac{1 - (1 + i)^{-n}}{i} \right]$</td>
<td>$\bar{A} = R(1 + i) \left( \frac{1 - (1 + i)^{-n}}{i} \right)$</td>
<td>$A_d = A(1 + i)^{-d}$</td>
</tr>
</tbody>
</table>

**EXERCISES**

1. Find the amount of a Php 100 ordinary annuity payable semi-annually for 3 years if money is worth 5% converted semi-annually.

2. A farmer bought a farming tractor. If it was purchased under the following terms: Php 50,000 down payment and Php 4,500 payment every month for 5 years. If money is worth 6% compounded monthly, find the cash price of the car.

3. Francis wants to accumulate a fund for his daughter’s graduation gift. He deposits Php 1,500 every 3 months for 9 years in a savings account that pays 4% compounded quarterly. How much would he save in the account at the end of 9 years? (assume that no withdrawals were made and the starting balance of the account is Php 0.00)

4. A student invests Php 500 every 6 months at 4% compounded semi-annually. Find his savings in 12 years.
5. Anika purchased a property and pays Php 250,000 cash and the balance is to be repaid in 20 annual payments of Php 10,000 each. If money is worth 4.5%, what is the cash value of the property?

6. Find the amount of an ordinary annuity which pays Php 850 at the end of each 3 months for 5 years if money is worth 8% compounded quarterly.

7. Every 6 months for 5 years, a father deposits Php 3000 in a trust company for his daughter’s education. If the money earns at 16% compounded semi-annually, how much will be in the fund after the 7th deposit? After the last deposit?

8. Mrs. Bautista is a chief financial officer of TCB Catering Services. He is proposed to the company that they offer a retirement plan for a company employee who is now 55 years of age. The plan will provide an annuity due of Php7,000 every year for 15 years upon retirement at the age of 65. The company is funding the plan with an annuity due of 10 years. If the rate of interest per year is 5%, what is the amount of installment that the company should pay to fund this retirement.

1.4 Stocks and Bonds

Stocks are also known are equity securities, while bonds are similar to promissory notes. These two financial instruments are commonly used by bankers, traders, and businessmen in their financial portfolios.

Stocks represent shares of ownership of a company. By a share, we mean a unit of ownership of a corporation’s profit and asset. For example, ABC Trading Co. has released 100 shares for its profit and assets. If Francis bought 50 shares from ABC Trading Co. then he has 50% ownership of the profits and assets of the company. People who buys stocks usually receives a certificate indicating the pertinent details of the stock like the company name, name of stock holder, certificate number, number of shares owned and par value of the share.

There are two types of stocks, the common stocks and the preferred stocks.
• **Common stock:** represents a share of company's asset and profit. Holders of common stock can vote in election of the board of directors (normally one vote per share). The board of directors oversee the management of the company, but do not directly run the company. Common stock is high risk and high return. Although common stocks yield higher return than other stocks, common shareholders stand to lose most when a company goes bankrupt.

• **Preferred stock:** Holders of preferred stock, in most cases, cannot vote. On the other hand, they are guaranteed a fixed dividend before any dividends are distributed to other shareholders. In the event of bankruptcy and liquidation, shareholders of preferred stocks are paid off after creditors and before common shareholders.

When a certain company sells stocks for the first time in public it is generally called an **IPO**, short term for initial public offering. In the literature, some refer to this as “Going Public”. The paper works and details of this public offering is being handled by financial institutions or investment banks called **Underwriters**. If for instance, an individual would like to buy all the shares of stocks that a company is selling, the individual should pay a **market cap** equivalent to the price per share multiplied to the number of shares being sold.

There are two ways of earning from stocks these happens when earnings are paid out to you in the form of dividends or there is an increase in share price.

Two types of stock market namely: Primary market and Secondary market. **Primary market** is where a company issues its shares for the first time via an IPO. **Secondary market** is commonly known as stock market where previously-issued stocks are traded without the involvement of the companies which issued them. Some well-known stock market include the New York Stock Exchange, NASDAQ, London Stock Exchange and Hong Kong Stock Exchange.

Here are some important tips when planning to invest in stocks.

- Invest in companies you understand and whose business make sense to you.
CHAPTER 1: MATHEMATICS OF INVESTMENT

- If you already bought some stocks or shares, stay for a while and do not expect to gain a lot in your first week or first month. Usually trading in stocks gives a reasonable return in the long term.

- Try not to panic when you see fluctuations in your stock values, but rather make informed decision.

- Once you have conceptualized an investment plan for yourself, you need to determine the amount that you want to invest. Check out the stocks you plan to acquire and set the timeframe you intend to keep your stock. And the important part is to follow your plan. However, asking for financial advice from credible analysts may also help you make a wise investment decision.

Corporation issue bonds to whoever wants to buy them. When you buy a bond, you are actually lending money to the corporation that issues the bond. The corporation in return, promises to pay interest on specified intervals for the length of the loan. In earlier times, bond certificates actually have printed coupon which can be detached to collect the interest. Physical possession of the certificate was proof of ownership.
There are several types of bonds. These are Government Bonds, Corporate Bonds and Zero-Coupon Bonds. **Government bonds** are bonds issued by the government to fund programs, meet payrolls, pay their bills or some economic strategy of the government. While **Corporate bonds** are bonds issued by business or companies to help them pay expenses or fund some company expansion program. These bonds are usually higher risk than government bonds but can earn more considerably. Another type of bonds is called **Zero-coupon bonds**. These bonds make no coupon payments but instead is issued at a considerable discount to par value.

Comparing stocks and bonds, we see that stocks are equities while bonds are thought of as debts or loans. When it comes to the growth of your money, stocks may have varying interest while
bonds are usually fixed as stated in the bond certificates or contracts. While stocks may give a higher yield rate compared to bonds, it is usually risky and unstable.

Bonds markets, unlike stock or share markets, often do not have a centralized exchange or trading system, while stock or share markets, have a centralized exchange or trading system. In the Philippines, the Philippine Stock Exchange located in Makati is the national stock exchange in the country.

Stocks pay dividends to the owners, but only if the corporation declares a dividend. Dividends are a distribution of a corporation's profits. Bonds pay interest to the bondholders. Generally, the bond contract requires that a fixed interest payment be made every six months.