A STUDY OF PATTERNS

The Nature of Mathematics and Mathematics in Nature
WHAT IS MATHEMATICS?

SOME SAY

- It is a study of numbers and arithmetic operations.
- It is a tool or a collection of skills that helps us answer question of “HOW MANY” or “HOW MUCH”.
- It is a science of logical reasoning, drawing conclusions from assumed premises or strategic reasoning.
- It is an art that studies patterns for predictive purposes or as a specialized language that deals with form, size and quantity.
What ever point of view we take, there is no denying the fact that Mathematics is universal.
GUESS WHAT’S NEXT..

1 * 1 = 1
11 * 11 = 121
111 * 111 = 12,321
1,111 * 1,111 = 1,234,321
11,111 * 11,111 = 123,454,321
111,111 * 111,111 = ????
What's the next shape?
WHAT’S THE NEXT SHAPE?
WHAT’S THE NEXT NUMBER?

1. 2, 4, 6, 8, 10, _____
2. 1, 3, 5, 7, 9, 11, ______
3. 1, 3, 7, 15, 31, ______
4. 448, 224, 112, 56, 28, _____
“A pattern is something which helps us anticipate what we might see or expect to happen next.”

“Patterns may also help know what may have come before what we are seeing currently.”
VARIOUS EXAMPLES OF PATTERNS

- Logic Patterns
- Number Patterns
- Geometric Patterns
- Word Patterns
PATTERNS AND ISOMETRIES

The Nature of Mathematics and Mathematics in Nature
Snow covers an orchard in the United States. "No orchard's the worse for the wintriest storm; but one thing about it, it mustn't get warm," wrote Robert Frost in his poem "Good-bye, and Keep Cold."

Photograph by Richard Olsenius
Lakeside Reflection

Photograph by Raymond Gehman

A still lake reflects sky and trees in Canada.
Scales from butterfly wings radiate from a glass-shelled diatom.

Photograph by Darlyne A. Murawski.
“A pattern has symmetry if there is an isometry of the plane that preserves the pattern.”
A Transformation is a process which shifts points of the plane to possibly new locations on the plane.
“A translation (or a slide) moves a shape in a given direction by sliding it up, down, sideways, or diagonally.”
A reflection (or a flip) can be thought of as getting a mirror image. It has a line of reflection or mirror line where the distance between the image and the mirror line is the same as that between the original figure and the mirror line.
“A rotation (or a turn) has a point about which the rotation is made and an angle that says how far to rotate.”
DILATION

“A dilation is a transformation which changes the size of an object”
RIGID TRANSFORMATIONS

“Transformations which leave the dimensions of the object and its image unchanged are called rigid transformations, or isometric transformations, or isometries.”
"An isometry of the plane is a mapping that preserves distance (and therefore shape):
\[ d(f(x), f(y)) = d(x, y) \]"

"iso" : Greek for "the same"

"metry/metria" : Greek for "measure"
COMBINATION OF ISOMETRIES

It is possible to combine isometries to produce other isometries.
GLIDE REFLECTION

A reflection followed by a translation or vice versa is called a glide reflection

http://www.wikiart.org/en/m-c-escher/horseman-1
PROPER AND IMPROPER ISOMETRIES

If an isometry is such that it cannot be applied without lifting the plane into space, it is called improper. Otherwise it is a proper isometry.

Any proper isometry is either a translation or a rotation, while an improper isometry is either a reflection or a glide reflection.
CHASLE’S THEOREM

Every motion of the plane is one of these transformations: a translation, a rotation, a reflection or a glide reflection
# Multiplication Table for Isometries

<table>
<thead>
<tr>
<th></th>
<th>Reflection</th>
<th>Translation</th>
<th>Rotation</th>
<th>Glide Reflection</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Reflection</strong></td>
<td>Translation or Rotation</td>
<td>Glide Reflection</td>
<td>Glide Reflection</td>
<td>Translation or Rotation</td>
</tr>
<tr>
<td><strong>Translation</strong></td>
<td>Glide Reflection</td>
<td>Translation</td>
<td>Rotation</td>
<td>Reflection or Glide Reflection</td>
</tr>
<tr>
<td><strong>Rotation</strong></td>
<td>Glide Reflection</td>
<td>Rotation</td>
<td>Translation or Rotation</td>
<td>Glide Reflection</td>
</tr>
<tr>
<td><strong>Glide Reflection</strong></td>
<td>Translation or Rotation</td>
<td>Reflection or Glide Reflection</td>
<td>Glide Reflection</td>
<td>Translation or Rotation</td>
</tr>
</tbody>
</table>
1. Answer the following exercises on page 292 of the book: #'s 1, 4, 13.